Subtraction Strategies

Students' strategies for subtraction fall into three basic categories: (1) Subtracting in parts, (2) Adding up or subtracting back from one number to the other, and (3) Changing the numbers to numbers that are easier to subtract. A fourth category, subtraction by place, is also described in this Teacher Note, but it is not emphasized in this unit. To use these strategies, students must understand the meaning of subtraction and have a good mental model of what is happening in the problem. They must be able to look at the problem as a whole, think about the relationships of the numbers in the problem, and choose an approach they can carry out easily and accurately.

At the end of this unit, fourth graders should be familiar with strategies in each of the first three categories. They should feel comfortable and confident with at least one strategy and should be able to use it efficiently—working with the largest or most reasonable parts of the number and using the fewest number of steps.

Here are examples of students' strategies for the following problem:

$$451 - 287 =$$

Subtracting in Parts

Cheyenne's strategy

451 − 200 = 251
251 − 80 = 171
171 − 7 = 164

Luke's strategy

451 − 100 = 351
351 − 100 = 251
251 − 50 = 201
201 − 30 = 171
171 − 7 = 164

Abdul's strategy

$$\begin{array}{r} 451 \\ -\ 200 \\ \hline 251 \\ -\ 50 \\ \hline 201 \\ -\ 30 \\ \hline 171 \\ -\ 7 \\ \hline 164 \end{array}$$

These three students subtracted 287 in parts. Cheyenne broke up 287 by place (200 + 80 + 7), and Luke subtracted one hundred at a time and then broke up the 87. Abdul subtracted 200 first and then broke 87 into 50 + 30 + 7. As students use this strategy, encourage them to subtract the largest parts they can while still making sense of the problem and the numbers. Work with students to gradually subtract larger amounts; for example, subtracting 80 rather than subtracting 50 and then 30. However, keep in mind that fluent students often quickly subtract smaller parts mentally without the need to write down all the steps. Students sometimes call this strategy "subtracting one number in parts."

Adding Up and Subtracting Back

In this category of strategies, students visualize how much more or less one number is than the other and either "add up" or "subtract back" to find their answer. They often represent the subtraction as the distance between two numbers on a number line.

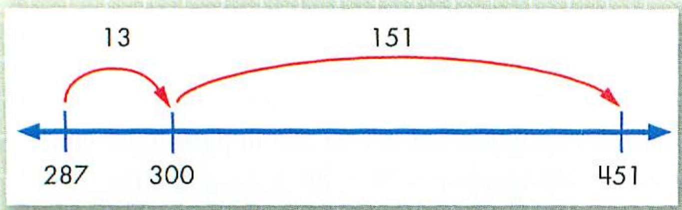
Set A, Adding up In Set A, students start at 287 and "add up" until they reach 451.

Sabrina's strategy

287 + $\underline{13}$ = 300
300 + $\underline{151}$ = 451
13 + 151 = 164

Derek's strategy

287 + $\underline{100}$ = 387
387 + $\underline{60}$ = 447
447 + $\underline{4}$ = 451
100 + 60 + 4 = 164



Both students thought of the solution as how much more must be added to 287 to get a sum of 451. Implicitly, they are using the inverse relationship of addition and subtraction to solve the problem. As shown on the number line, Sabrina added 13 to 287 to get to 300 and then added 151 to get to 451. Derek looked for the largest multiple of 100 and then the largest multiple of 10. Students often call this strategy "adding up."

Set B, Subtracting back In this set of solutions, students started at 451, and then subtracted back until they reached 287.

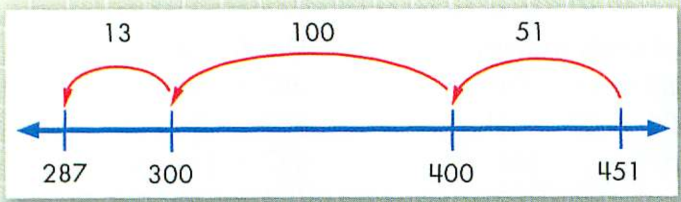
Bill's strategy

451 − 51 = 400
400 − 100 = 300
300 − 13 = 287

51 + 100 + 13 = 164

Yuki's strategy

451
− 151
300
− 13
287
151 + 13 = 164



Both students solved the problem by "going back" to 300 (Bill took two steps, Yuki did it in one) and then "back 13 more" to 287. As you can see in the number line representation, students are not subtracting 287 in parts as in the first category; rather, they start at 451, subtract until they reach 287, and then determine how much they subtracted. Students often describe this method as figuring out "how far" 287 is from 451.

Changing the Numbers

In this group of strategies, students change one or both of the numbers to what they often call "landmark" or "friendly" numbers.

Set A, Changing and compensating In Set A, students changed one or both of the numbers, subtracted, and then compensated for the changes they had made.

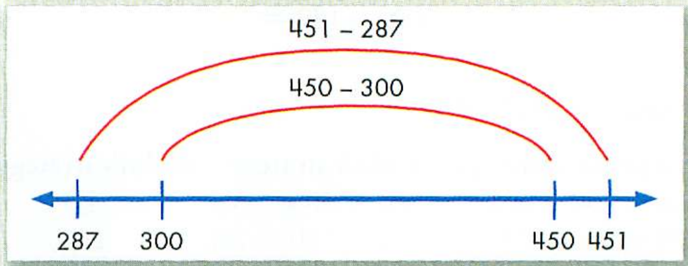
LaTanya's strategy

451 − 300 = 151
151 + 13 = 164

Venetta's strategy

450 − 300 = 150
150 + 14 = 164

LaTanya changed 287 to 300 to create an easier subtraction problem. Because she had subtracted 13 too much, 13 was then added to 151 to get the final answer. Venetta changed both numbers and had to decide how both those changes affected the result. The difference between the two numbers was decreased by 1 (changing the 451 to 450) and by 13 (changing the 287 to 300), so 14 is added to 150. Visualizing the effect of the changes and how to compensate for those changes is critical to this kind of strategy. Number lines are particularly useful tools for visualizing how changing numbers affects the result. In this case, both changes made the difference smaller.



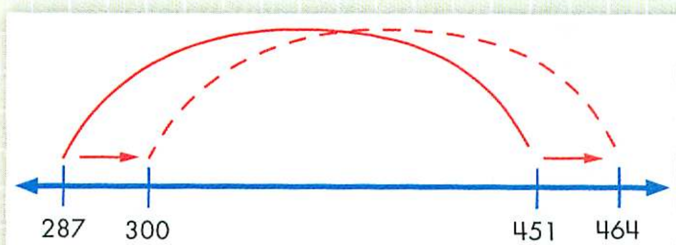
In general, we do not encourage students to change both numbers in a subtraction problem (as in Venetta's example of changing 451 − 287 to 450 − 300, subtracting, and then adjusting the result). If a student can indeed visualize what these changes mean and how to adjust the result to get the answer to the original problem, then that student can certainly use this method when it makes sense for a particular problem. However, too many students change both numbers to "easy" numbers without a clear idea of how those changes affect the difference. Classroom experience indicates that thinking through how to adjust the result after changing only one number in a subtraction problem can be challenging for Grade 4 students. Discussions of this idea (as in Session 4.4) can be very fruitful in helping students think more deeply about the operation of subtraction and the relationship between the two numbers in a subtraction expression, whether or not students actually use this method to solve problems.

Set B, Making an equivalent problem The next student's strategy is an example of changing both numbers in order to create an equivalent problem that can then be solved without any need to compensate for changes.

Marisol's strategy

$287 + 13 = 300$
$451 + 13 = 464$
$451 - 287 = 464 - 300 = 164$

Marisol created an equivalent problem that is easier to solve by adding 13 to both numbers. Adding, or subtracting, the same quantity to or from both numbers maintains the difference. You may visualize transforming the problem in this way as sliding the difference along a number line.



Although it is not surprising for students to use this strategy, it is not as common as the others listed here. It is not explicitly mentioned in the text of Investigation 4, but it may come up in your classroom.

Subtracting by Place

Adding by place—adding 1s to 1s, 10s to 10s, 100s to 100s, and so on—is one of the addition strategies most often used by students. However, subtracting by place is not as straightforward. Consider the problem $451 - 287$. It is easy to subtract 200 from 400, but how do you subtract 80 from 50 or 7 from 1? The following two strategies are based on subtracting by place value. Jake's is the "borrowing," or regrouping, algorithm, which has been commonly taught in the United States. This algorithm requires recomposing the number 451 to make it possible to subtract in each place. The shorthand notation shown below means that 451 $(400 + 50 + 1)$ has been recomposed into $300 + 140 + 11$, which then allows easy subtraction by place. (This algorithm will be studied in Grade 5.) Alejandro uses a strategy often developed by students who understand that it is possible to subtract, for example, 80 from 50, resulting in -30. Using this method, subtracting by place results in differences for each place, which can be positive or negative. Combining these differences gives the answer to the problem.

Jake's strategy

$$\begin{array}{r} {}^{3}\not{4}\,{}^{14}\not{5}\,{}^{1}1 \\ -\ 2\ 8\ 7 \\ \hline 1\ 6\ 4 \end{array}$$

Alejandro's strategy

$$\begin{array}{r} 451 \\ -\ 287 \\ \hline 200 \\ -\ 30 \\ \hline 170 \\ -\ 6 \\ \hline 164 \end{array}$$

Neither of these strategies is mentioned explicitly in the Investigation.